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SIMPLE SIMULATIONS OF MASSLESS SCALAR FIELD MINIMALLY COUPLED TO GRAVITATIONAL FIELD

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Abstract

Simple simulations have been implemented for Massless Scalar Field Minimally Coupled to Gravitational Field within the framework of general relativity and astrophysics. Distinct features of 1-D Einstein-Klein-Gordon (EKD) system have been utilized and interesting simulations for physical properties have been carried out.

Keywords: Massless scalar field, 1-D Einstein-Klein-Gordon system

Introduction

Black hole spacetimes are spacetimes possessing remarkable properties. The four laws of black hole mechanics and their relation to the familiar thermodynamical ones suggest a deep connection between gravity, quantum physics, and thermodynamics a connection that despite persistent efforts still has not been fully understood (Alan R. Parry,2012). If the Cosmic Censorship Hypothesis (V. F. Mukhanov, 2000) holds true.

Black hole spacetime is not an exclusive property of the vacuo or electrovacuo Einstein's equations. Einstein's gravity coupled to scalar or other fields admits called hairy family of black holes. However, here matters are not yet on a firm state as for the vacuum or electrovacuum case. Even though a number of uniqueness theorems for particular field configurations have been established,(R.Dave,1998), their classification is for the moment open and mathematically challenging problems are likely to persist for some time . The absence of black hole states within the Einstein- Maxwell system possessing an ergoregion disconnected from the event horizon has been settled only relatively recently with the work of Sudarsky and Wald.

In this work we shall assume that the external matter is endowed with a scalar charge distribution so that the same spacetime region is permeated by a massless scalar field Φ obeying the Einstein-Klein-Gordon equations. We shall show that such a region can be described by a spacetime (M, g, Φ) with

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(g,Φ) particular static axisymmetric solutions of the Einstein-Klein-Gordon system admitting an isometric extension into a larger manifold (M',g',Φ') possessing a regular $R\times S^2$ bifurcating Killing horizon. This bifurcating regular Killing horizon may be elevated to the status of an event horizon and thus (M',g',Φ') may be interpreted as representing the spacetime near the event horizon of a distorted black hole of the Einstein-Klein-Gordon system. In this work we construct two families of such solutions characterized by an infinite set of parameters interpreted as describing the structure of the perturbing distribution of matter as well as the structure of the scalar charge distribution responsible for the field Φ.

Static-Axisymmetric Solutions of the Einstein-Klein-Gordon Equations, Admitting a Bifurcating Killing Horizon

The Einstein-Klein-Gordon minimally coupled to gravity field equations on a spacetime *M* are described by

$$G_{\mu\nu}=k(\nabla_\mu\Phi\nabla_\nu\Phi-\frac{1}{2}g_{\mu\nu}\nabla^\sigma\Phi\nabla_\sigma\Phi), \qquad (1)$$

$$\nabla_\mu\nabla_\nu\Phi=0 \qquad (2)$$

For any nonsingular, minimally C^2 solution (g,Φ) of the above equations admitting two hyper surfaces orthogonal, commuting, and orthogonal Killing vector fields, a timelike one ξ_t and a spacelike ξ_φ chart (t,φ,x^1,x^2) can be constructed so that $\xi_t=\frac{\partial}{\partial t}$, $\xi_\varphi=\frac{\partial}{\partial\varphi}$, and g takes the Weyl canonical form:

$$g=-e^{2U}dt^2+r^2e^{-2U}d\varphi^2+e^{2(V-U)}(dr^2+dz^2), \qquad (3)$$

where $U=U(r,z)$, $V=V(r,z)$, and points on the manifold where $r=0$ define the symmetry axis associated with the rotational Killing field ξ_φ. Relative to this local chart, equations (1) and (2) implies that $U=U(r,z)$, $\Phi(r,z)$ and $V=V(r,z)$, satisfy

$$\frac{\partial^2 U}{\partial r^2}+\frac{\partial^2 U}{\partial z^2}+\frac{1}{r}\frac{\partial U}{\partial r}=0, \tag{4}$$

$$\frac{\partial^2 \Phi}{\partial r^2}+\frac{\partial^2 \Phi}{\partial z^2}+\frac{1}{r}\frac{\partial \Phi}{\partial r}=0, \tag{5}$$

$$\frac{\partial V}{\partial r}=r\left\{\left(\frac{\partial U}{\partial r}\right)^2+\left(\frac{\partial U}{\partial z}\right)^2+\frac{k}{2}\left[\left(\frac{\partial \Phi}{\partial r}\right)^2-\left(\frac{\partial \Phi}{\partial z}\right)^2\right]\right\}, \tag{6}$$

$$\frac{\partial V}{\partial z}=r\left[2\frac{\partial U}{\partial r}\frac{\partial U}{\partial z}+k\frac{\partial \Phi}{\partial r}\frac{\partial \Phi}{\partial z}\right]. \tag{7}$$

Equations (4) and (5) are recognized as the Laplace equations on Euclidean R^3 equipped with cylindrical coordinates $\Phi(r,z)$ and once a choice of the harmonic functions (U,Φ) has been made, equations (6) and (7) determine V via quadratures. The integrability conditions for the existence of V are satisfied by virtue of equations (4) and (5).

Our goal is to construct solutions of equations (4) and (7) so that the resulting (g,Φ) admits an extension possessing a regular bifurcating Killing horizon. In order to identify such solution we consider an open subset $S=\{(r,z,\Phi)|r<a,\ z<b\ ,ab\neq 0\}$ containing the origin of the Euclidean three space, where (r,z,Φ) are standard cylindrical coordinates and (a,b) arbitrary for the moment parameters. We then consider the product manifold $M\times R\times S$ and for any triplet (U,V,Φ) satisfying equations (4), (7) on S, we take their lifts on $M\times R\times S$. In view of equation (3) they define a static-axisymmetric metric g and a scalar field on $M\times R\times S$, satisfying the covariant equations (1) and (2). At first we require that the solutions (U,V,Φ) of equations(4) and (7) must be chosen so that the resulting (g,Φ) on $M=R\times S$ on (U,V,Φ) would satisfy the following:

(α) elementary flatness holds true on any point of the axis;

(β) the spacetime ($R\times S, g, \Phi$)is singularity free.

A point of departure for the specification of such a triplet is the observation that any regular axisymmetric harmonic functions, say, standing for field Φ on *S* can be represented in the form:

$$\Phi(r,z) := \sum_{l=0}^{\infty} \alpha_l (r^2+z^2)^{l/2} P_l\left(\frac{z}{\sqrt{r^2+z^2}}\right), \tag{8}$$

where $\alpha_l,\ l=0,1,\ldots$ are arbitrary constants while P_l stands for the Legendre polynomials. We begin applying the above procedure by taking Φ as above and choosing for *U* the trivial harmonic function $U \equiv 0$. Despite this special choice, the function *V* resulting from the integration of equations (6) and (7) is rather complicated. For simplicity we shall employ hereafter a truncated version of equation (8) described by

$$\Phi(r,z) = \alpha_0 + \alpha_1 z + \alpha_2(2z^2 - r^2). \tag{9}$$

Making use of this $\Phi(r,z)$, the integration of equations (6) and (7) combined with $U \equiv 0$ yields

$$V(r,z) = \frac{1}{4}kr^2[2\alpha_2^2 r^2 - (4\alpha_2 z + \alpha_1)^2] + V_0. \tag{10}$$

By setting $V_0 = 0$ it follows immediately that $V(r,z)$ is vanishing on the entire *z* axis. The so-constructed $U(r,z) \equiv 0$ combined with the metric

$$g = -dt^2 + r^2 d\phi^2 + e^{2V(r,z)}(dr^2 + dz^2). \tag{11}$$

which is a static, axially symmetric metric admitting $\frac{\partial}{\partial t}$ as a timelike Killing vector field possessing complete orbits, commuting with the axial Killing field $\frac{\partial}{\partial \varphi}$, and both fields are hypersurface orthogonal. By virtue of the fact that $V(r=0,z)=0$ it is regular on the axis and moreover it can be checked directly that this *g* combined with described by equation (9) satisfies the covariant equations (1) and (2). A computation of the scalar invariants $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$, $C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}$, $R^{\alpha\beta}R_{\alpha\beta}$, and *R* yields

$$R^{\mu\nu\sigma\tau}R_{\mu\nu\sigma\tau} = 4C^{\mu\nu\sigma\tau}C_{\mu\nu\sigma\tau} = 3R^{\mu\nu}R_{\mu\nu} = 3R^2 \tag{12}$$

$$R = [4\alpha_2^2(r^2+4z^2)+8\ \alpha_1\alpha_2 z+\alpha_1^2]e^{f(r,z)}, \tag{13}$$

$$f(r,z) = \frac{1}{2}kr^2[2\alpha_2^2(8z^2-r^2)+\alpha_1(8\ \alpha_2 z+\alpha_1)],$$

indicating a regular geometry on any point on $M = R\times S$. Extending the coordinates (r,z) over the entire plane, the resulting spacetime $(M = R\times R^3, g, \Phi)$ fails to be asymptotically flat and thus equations (9) and (11) appear to be of limited utility. However, by appealing to the partial linearity afforded by equations (4) and (7), they can used as a seed for the construction of more interesting families of staticaxisymmetric solutions of equations(1) and (2). In that regard we recall that the positive mass Schwarzschild metric in Weyl coordinates e(3) is described by

$$U_{BH} = \frac{1}{2}\ln\left(\frac{R_+ + R_- - 2m}{R_+ + R_- + 2m}\right)$$

$$V_{BH} = \frac{1}{2}\ln\left(\frac{(R_+ + R_-)^2 - 4m^2}{4R_+R_-}\right) \tag{14}$$

$$R_\pm^2 = r^2 + (z\pm m)^2,$$

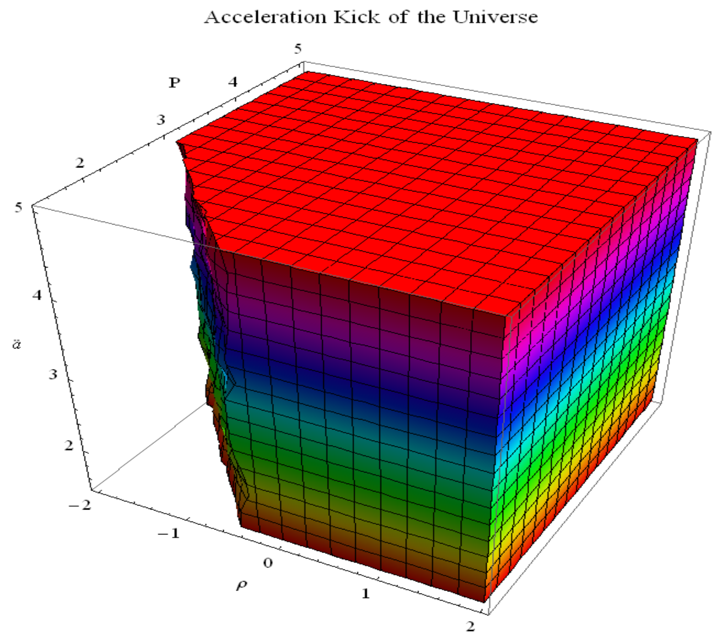

Figure 1: Occupied Acceleration Kick of the Universe

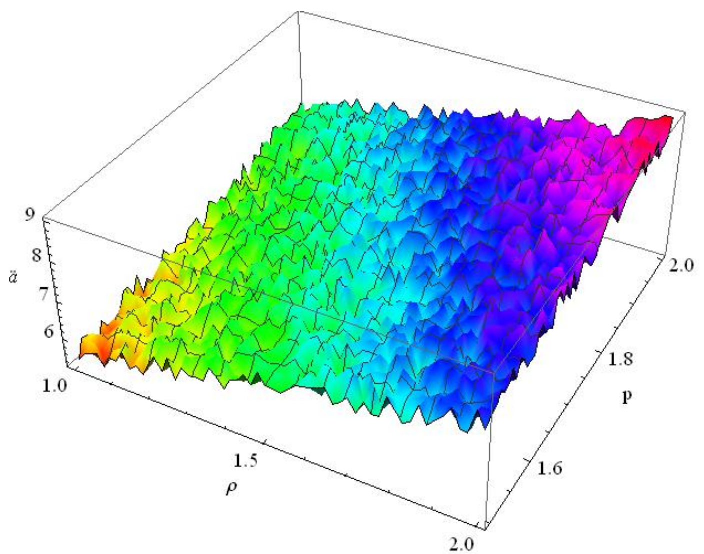

Figure 2: Formal development profile of $\ddot{a}$ in terms of pressure and energy density ρ*

Concluding Remarks

To make this discussion precise, we need a model for the regular matter. In order to study stability questions, we need to know how the regular matter distribution changes as the wave dark matter distribution changes, and vice versa. For example, a relatively simple way to model regular matter is with another scalar field. There are others ways to model regular matter which we do not discuss here. We caution the reader that this second scalar field is only a practical device for approximately modeling the regular baryonic matter namely the gas, dust, and stars in a galaxy. In no way are we suggesting a second scalar field should exist physically. Furthermore, the parameters of this second scalar field are chosen simply to fit the regular matter distribution of a galaxy as well as possible.

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